Math 1411 Quiz 10 Name KEY

You may use a calculator and your homework, but not your books or notes. There are three problems worth 10 points each. **Show all of your work to receive full/partial credit.**

1) Find the indefinite integral and check the results by differentiation.

$$\int \frac{x^3}{\sqrt{1+x^4}}\,dx$$

$u = 1+x^4$
$du = 4x^3dx$
$\frac{1}{4}du = x^3dx$

$\frac{1}{4}\int \frac{du}{\sqrt{u}} = \frac{1}{4}\int u^{-1/2}du$

$= \frac{1}{4}(2u^{1/2}) + C = \frac{1}{2}(1+x^4)^{1/2} + C$

Check: $\frac{d}{dx}\left[\frac{1}{2}(1+x^4)^{1/2} + C\right] = \frac{1}{4}(1+x^4)^{-1/2}(4x^3) = \frac{x^3}{\sqrt{1+x^4}}$

2) (#11 from 4.6) Approximate the definite integral using **Simpson's Rule** with $n = 4$. Compare the result with the approximation of the integral using a graphing utility.

$$\int_0^2 \sqrt{1+x^3}\,dx$$

$n=4$ means $x_0 = 0$, $x_1 = \frac{1}{2}$, $x_2 = 1$, $x_3 = \frac{3}{2}$, $x_4 = 2$

$\frac{b-a}{3n}\left[f(x_0)+4f(x_1)+2f(x_2)+\cdots+f(x_4)\right] = \frac{2}{12}\left[f(0) + 4f(\tfrac{1}{2}) + 2f(1) + 4f(\tfrac{3}{2}) + f(2)\right]$

$= \frac{1}{6}\left[1 + 4\left(\frac{3\sqrt{2}}{4}\right) + 2\sqrt{2} + 4\left(\frac{\sqrt{70}}{4}\right) + 3\right]$

$= \frac{1}{6}\left[\sqrt{70} + 5\sqrt{2} + 4\right] \simeq 3.23961$

(Approximately 3.24131 according to TI-89)

3) (#91 from 5.1) Use logarithmic differentiation to find dy/dx.

$$y = \frac{x^2\sqrt{3x-2}}{(x+1)^2}, \quad x > \frac{2}{3}$$

$$\ln y = \ln\left[\frac{x^2\sqrt{3x-2}}{(x+1)^2}\right] = 2\ln x + \frac{1}{2}\ln(3x-2) - 2\ln(x+1)$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{1}{2}\left(\frac{3}{3x-2}\right) - \frac{2}{x+1}$$

$$y' = y\left[\frac{2}{x} + \frac{3}{6x-4} - \frac{2}{x+1}\right]$$

$$y' = \frac{x^2\sqrt{3x-2}}{(x+1)^2}\left(\frac{2}{x} + \frac{3}{6x-4} - \frac{2}{x+1}\right)$$